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الف)

$(x+y^2)\,dx-2xy\,dy=0$

$M(x,y)=x+y^2 \qquad M_y=2y$

$N(x,y)=-2xy \qquad N_x=-2y$

$\boxed{\neq \text{ کامل نیست}}$

$g(x)=\dfrac{M_y-N_x}{N}=\dfrac{2y-(-2y)}{-2xy}=\dfrac{4y}{-2xy}=-\dfrac{2}{x}$

$\mu(x)=e^{\int g(x)\,dx}=e^{\int -\frac{2}{x}dx}$

$=e^{-2\int \frac{1}{x}dx}=e^{-2\ln x}=e^{\ln x^{-2}}=x^{-2}$

$x^{-2}\left((x+y^2)\,dx-2xy\,dy\right)=0$

$(x^{-1}+x^{-2}y^2)\,dx-2x^{-1}y\,dy=0$

$M=x^{-1}+x^{-2}y^2 \qquad M_y=2yx^{-2}$

$N=-2x^{-1}y \qquad N_x=2x^{-2}y$

$\boxed{= \text{ کامل است}}$

$\int x^{-1}+x^{-2}y^2\,dx=\int 2x^{-1}y\,dy$

$\ln|x|+-1\dfrac{1}{x}y^2-2x^{-1}\dfrac{y^2}{2}+C$

۱- ب)

$(x^2+y^2)\,dx + 9xy\,dy = 0$

$M(x,y) = x^2+y^2 \qquad M_y = 2y$

$N(x,y) = 9xy \qquad N_x = 9y \qquad \neq$ کامل نیست

$g(x) = \frac{2y - 9y}{9xy} = \frac{-7y}{9xy} = -\frac{7}{9x}$

$\mu(x) = e^{\int g(x)\,dx} = e^{\int -\frac{7}{9}\times\frac{1}{x}} = e^{-\frac{7}{9}\ln x} = e^{\ln x^{-\frac{7}{9}}} \qquad x^{-\frac{7}{9}}$

$x^{-\frac{7}{9}}\left((x^2+y^2)\,dx + 9xy\,dy\right) = 0$

$\boxed{-\frac{7}{9}+\frac{2}{1} = \frac{-7+18}{9} = \frac{11}{9}}$

$(x^{\frac{11}{9}} + x^{-\frac{7}{9}}y^4)\,dx + 9x^{\frac{2}{9}}y\,dy = 0$

$\boxed{-\frac{7}{9}+1 = \frac{2}{9}}$

$M(x,y) = x^{\frac{11}{9}} + x^{-\frac{7}{9}}y^2 \qquad M_y = 2yx^{-\frac{7}{9}}$

$N(x,y) = 9x^{\frac{2}{9}} \qquad N_x = 2x^{-\frac{7}{9}} \qquad =$ کامل است

$\int (x^{\frac{11}{9}} + x^{-\frac{7}{9}}y^2)\,dx = -\int 9x^{\frac{2}{9}}y\,dy$

$\frac{x^{\frac{20}{9}}}{\frac{20}{9}} + \frac{x^{\frac{2}{9}}}{\frac{2}{9}}y^2 = -9x^{\frac{2}{9}}\frac{y^2}{2}$

$\frac{9}{20}x^{\frac{20}{9}} + 9x^{\frac{2}{9}}y^2 + 9x^{\frac{2}{9}}y^2 + C$

$$(x^2+1)y'' - 2xy' + 2y = 9(x^2+1)^2$$ (ج-

$$\div (x^2+1)$$

$$y'' - \frac{2x}{(x^2+1)} y' + \frac{2}{(x^2+1)} y = 9(x^2+1)$$

$$y'' - \frac{2x}{(x^2+1)} y' + \frac{2}{(x^2+1)} y = 0 \qquad y_1 = x$$ جواب خصوصی

$$v = \int \left(\frac{1}{x^2} e^{\int \frac{2x}{x^2+1} dx} \right) dx$$

$$\int \frac{x^2+1}{x^2} dx = x - \frac{1}{x} \qquad y_2 = x^2 - 1$$ جواب دوم

$$y_p = c_1 x + c_2 (x^2-1)$$

$$y_p = v_1 x + v_2 (x^2-1)$$

$$v_1' x + v_2'(x^2-1) = 0$$

$$v_1' + 2v_2' x = 9(x^2+1)$$

$$v_1' = -9(x^2-1) \qquad v_2' = 9x$$

$$v_1 = \int -9(x^2-1)\, dx \qquad v_2 = \int 9x$$

$$v_1 = -9\frac{x^3}{3} + 9x \qquad v_2 = 9\frac{x^2}{2} \text{ or } \frac{9}{2}x^2$$

$$y_p = \left(-9\frac{x^3}{3} + 9x\right)x + \frac{9}{2}x^2(x^2-1)$$

$$y = c_1 x + c_2(x^2-1) + \left(-\frac{9}{3}x^3 + 9x\right)x + \frac{9}{2}x^2(x^2-1)$$

$$y' - \frac{y}{2x} = -y^5 \quad \div (-y^5) \qquad\qquad (> -1$$

$$-y^{-5}y' + y^{-4} \times \frac{1}{2x} = 1$$

$$u = y^{-4} \qquad u' = -4y^{-5}y'$$

$$\frac{1}{4}u' + \frac{1}{2x}u = 1$$

$$M'(x = e^{\int \frac{1}{2x} dx} = e^{\frac{1}{2}\int \frac{1}{x} dx} = e^{\frac{1}{2}\ln|x|} = e^{\ln x^{\frac{1}{2}}} = x^{\frac{1}{2}}$$

$$u = x^{-\frac{1}{2}} \left[\int 1 (x^{\frac{1}{2}}) \, dx + C \right]$$

$$u = x^{-\frac{1}{2}} \times \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + Cx \Rightarrow u = x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + Cx$$

$$\text{so} \Rightarrow u = y^{-4}$$

$$\Rightarrow x^{-\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + Cx = y^{-4}$$

$$y = \sqrt[4]{\frac{1}{x^{-\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + Cx}}$$

$y'' - ky' - 6y = e^x \sin 2x$ ۱-ز)

$y'' - 9y' - 6y = e^x \sin 2x$

$\lambda^2 - 9\lambda - 6 = 0$

$\Delta = b^2 - 4ac = 81 - 4 \times 1 \times -6$

$81 + 24 = 105$

$\lambda = \dfrac{9 \pm \sqrt{105}}{2}$ $\quad \lambda_1 = \dfrac{9 + \sqrt{105}}{2}$

$\lambda_2 = \dfrac{9 - \sqrt{105}}{2}$

$y_p = c_1 e^{\frac{9 + \sqrt{105}}{2}} + c_2 e^{\frac{9 + \sqrt{105}}{2}}$ جواب عمومی

$y_p = A e^x \sin x + B e^x \cos x$

$y'_p = A e^x \sin x + A e^x \cos x + B e^x \cos x - B e^x \sin x$

$y''_p = \cancel{A e^x \sin x} + A e^x \cos x + A e^x \cos x - \cancel{A e^x \sin x} + \cancel{B e^x \cos x}$

$- B e^x \sin s - B e^x \sin x - \cancel{B e^x \cos x}$

$= 2 A e^x \cos x - 2 B e^x \sin x$

$y'' - 9y' - 6y = e^x \sin 2x$

$2 A e^x \cos x - 2 B e^x \sin x - 9 A e^x \sin x - 9 A e^x \cos x - 9 B e^x \cos x$

$+ 9 B e^x \sin x - 6 A e^x \sin x - 6 B e^x \cos x = e^x \sin 2x$

$- 6 A e^x \cos x - 16 A e^x \sin x + 6 B^x \sin x - 16 B e^x \cos x$

$A e^x (-6 \cos x - 16 \sin x) + B e^x (6 \sin x - \cos x)$

$y(x) = c_1 e^{\frac{9 - \sqrt{105}}{2}} + c_2 e^{\frac{9 + \sqrt{105}}{2}} + A e^x (6 \cos x - 16 \sin x)$

$+ B e^x (6 \sin x - \cos x)$

۱-ه)

$$y''' - \frac{k}{x} y'' = 0$$

$$y''' - \frac{9}{x} y'' = 0$$

$$u = y'' \longrightarrow u' = y'''$$

$$u' - \frac{9}{x} u = 0 \qquad u' = \frac{9}{x} u \qquad \int u' \, du = \int \frac{9}{x} u$$

$$u(x) = C_1 x$$

$$y'' = C_1 x$$

$$\int y'' \, dy = \int C_1 x \, dx$$

$$y' = \frac{1}{2} C_1 x^2 + C_2$$

$$\int y' \, dy = \int \left(\frac{1}{2} C_1 x^2 + C_2 \right) dx$$

$$y = \frac{1}{6} C_1 x^3 + C_2 x + C_3$$

$$y = C_1 x^3 + C_2 x + C_3$$

$y'' - y' = 9x - 1$ (9-1

$\lambda^2 - \lambda = 0 \quad \lambda(\lambda - 1) = 0 \quad \lambda = 0, 1$

$y_p = x(A_0 + A_1 x) = A_0 x + A_1 x^2$

$y'_p = A_0 + 2A_1 x \Rightarrow y''_p = 2A_1$

$y'' - y' = 9x - 1$

$2A_1 (-A_0 + 2A_1 x) = 9x - 1$

$2A_1 - A_0 = -1 \Rightarrow -9 - A_0 = -1 \quad A_0 = -8$

$-2A_1 = 9 \Rightarrow A_1 \left(\frac{-9}{2}\right) = -4{,}5$

$y_p = A_0 x + A_1 x^2$

$y_p = -8x - \frac{9}{2}x^2$

سوال ۲)

$x^2y'' + 4xy' - 4y = 0$ $\qquad$ $y_1 = x$

$y_1' = 1$

$y_1'' = 0$

$\cancel{x^2(0)} + 4x(1) - 4(x) = 0$

$0 + 4x - 4x = 0$ $\quad$ پس $y_1 = x$ جواب مسئله

$x^2y'' + 4xy' - 4y = 0 \quad (\div x^2)$

$y'' + \frac{4}{x}y' - \frac{4}{x^2}y = x$

$y = y_1 v = xv$

$y' = v + xv'$

$y'' = 2v' + xv'' \Rightarrow y'' + \frac{4}{x}y' - \frac{4}{x^2}y = x$

$(2v' + xv'') + \frac{4}{x}(v + xv') - \frac{4}{x^2}(xv)$

$2v' + x' + \frac{4}{x}v + 4v' - \frac{4}{x}v = 0$

$6v' + xv'' = x$

$w = v' \qquad 6w + xw' = x$

$w' = v''$

$w = x^{-\int x\,dx} \quad w = -\frac{x^7}{2} + c_1$

$e^{-\frac{x^7}{2}} \int x \cdot x\,dx + c = \frac{x^2}{7} + \frac{c_1}{x}$

$w = v' = \frac{x^2}{7} + \frac{c_1}{x}$

$t x''(t) - 4t x'(t) - 4x(t) = 0 \qquad x(0) = 0$ (3

$x'(0) = 1$

$\div t$

$x''(t) - 4x'(t) - 4x = 0$

$L\{(x''(t)\} (-4) L\{x'(t)\} - 4 L\{x(t)\}$

$= L\{0\} = 0$

$s^2 x(x - s \cancel{x^0(0)} - \cancel{x'^1(0)} - 4 s x(s)$

$- \cancel{4 x_0^0(0)} - 4 x(s) = 0$

$(s^2 - 4s + 4) \; x(s) = 0$

$x(s = \frac{1}{s^2 - 4s + 4} = \frac{1}{(s-2)(s-2)} = \frac{1}{(s-2)^2}$

$x(s = \frac{1}{s-2} - \frac{1}{s+2}$

$x(t) = L^{-1} \left\{ \frac{1}{s+2} - \frac{1}{s-2} \right\}$

$e^{+2it} - e^{-2t}$

(5)

$$\frac{dx}{dt} = 4x + 2y$$

$$\frac{dy}{dt} = -x + y$$

$$L\left\{\frac{dx}{dt}\right\} = 4L\{x(t)\} + 2L\{y(t)\}$$

$$L\left\{\frac{dy}{dt}\right\} = -L\{x(t)\} + L\{y(t)\}$$

$$L\{x(t)\} = x(s)$$

$$L\{y(t)\} = y(s)$$

$$sx(s) - x(0) = 4x(s) + 2y(s)$$

$$sy(s) - y(0) = -x(s) + y(s)$$

$$x(s)(s-4) = 2y(s) + x(0)$$

$$\cancel{x(s)(s-4) = 2y(s) + x(0)}$$

$$x(s) = \frac{2y(s) - x(0)}{s-4}$$

$$(s-1)\,y(s) = -x(s) + y(0)$$

$$y(s) = \frac{-x(s) - y(0)}{s-1}$$